SIGNIFICANT FIGURES/DIGITS

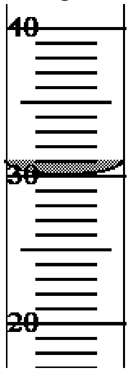
Definitions:

- **A DIGIT is any positive integer (i.e., whole number): 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.**
- **A FIGURE is a numerical value that is expressed using digits.**

What makes a figure "significant"?

To understand the concept of a significant figure, you must first understand **<u>why</u>** we use them.

Take this graduated cylinder, for example:



If asked to express the volume of liquid in the graduated cylinder, the answer would not be as straight forward as you might think.

- Is it 30 mL?
- Is it 31 mL
- Is it 30.1 mL?
- How about 30.02 mL?
- Or 29.9 mL?
- Or even 31 mL?

If you look at the ***bottom of the meniscus***, you will see that the liquid appears to be on the 30 mL line.

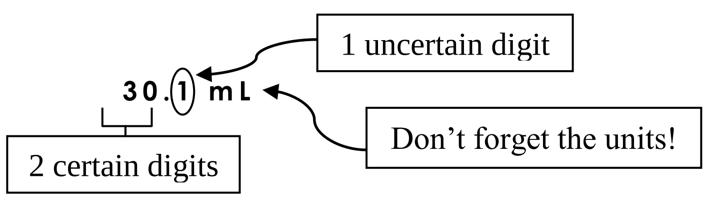
> - This does not mean, however, that this graduated cylinder contains EXACTLY 30 mL of liquid.

The equipment we use restricts the accuracy (how close we can come to measuring the ACTUAL value) of our measurements. Upon closer inspection, you will see that this graduated cylinder measures volume in increments of 1 mL.

- If you were to read a volume from this graduated cylinder, then, you could only be CERTAIN of your measurement down to 1 mL.
- You have no way of knowing if the volume is 30 mL, 30.5 mL, or even 30.0004 mL. The only two digits you can be absolutely certain of are the 3 and the 0.

Whenever reading data from an analog instrument like a graduated cylinder (as opposed to a digital instrument like an electronic balance), you MUST **<u>record ONLY the digits you are certain of PLUS</u> one additional digit that you estimate.**

- In this case, the "3" and the "0" are the digits that you are certain of, PLUS one extra digit called the UNCERTAIN number – because it's just an estimate or a guess.
- Because the meniscus looks like it lies RIGHT on the line at 30 mL, you can estimate the uncertain digit to be 0, or 1, or whatever you feel is right. In this case, we will say that the graduated cylinder is measuring a volume of:



All certain digits and the 1 uncertain digit are considered **SIGNIFICANT.** A significant figure is important because the accuracy of any values that we report is limited by the number of significant digits in our measurement - 3 SIGNIFICANT DIGITS in this case. No more, no less!

How do we recognize which digits are significant?

- RULE #1: All non-zero digit is significant (1, 2, 3, 4, 5, 6, 7, 8, 9).
- RULE #2: Any leading zeroes (to the left of a non-zero) are NOT significant.
- RULE #3: Any zero in-between non-zero digits (regardless of how far apart) is significant.
- RULE #4: Any zero to the right of a non-zero is only significant if it is also to the right of a decimal place.

For Example,

i. 10 has 1 significant digit
ii. 1010 has 3 significant digits
iii. 0.001 has 1 significant digit
iv. 0.001010 has 4 significant digits

PRACTICE - TRY SOME FOR YOURSELF

(a) 41.233 has ______ significant digits
(b) 0.057 has ______ significant digits
(c) 904.2 has ______ significant digits
(d) 8.10 has ______ significant digits
(e) 3.0100 has ______ significant digits
(f) 800,500 has ______ significant digits

SPECIAL EXCEPTIONS:

- **Counted Values** – usually something that you count (like apples, people, books, or atoms in a compound)
  - By counting these, you know **exactly** how many you have and, therefore, can be infinitely certain of their accuracy.
  - For example, if you have only 2 apples in your hand or 3 atoms in a molecule of carbon dioxide (CO_2 in case you forgot!) that would represent exactly 2 apples, not 2.5 and not 1.95.
  - Therefore every counted value has an **infinite number of significant digits**.
- **Constants** – these values that have been determined by someone typically have many significant digits that can be easily determined. However, in order for our class to maintain consistency and attempt to get the same answer, we will limit the number of digits that we use, but not the number's significance.
  - For example, Pi (π) has been determined well beyond the 3.14159265 (refer to: http://www.math.utah.edu/~pa/math/pi.html if you care that much!). For our class, we will ONLY **use Pi as 3.14 but treat it as infinitely significant.**
- **Numbers in a Formula** – occasionally we will see integer values in a mathematical formula, such as the circumference of a circle ($C = 2 \pi r$). This value includes the number 2, "a doubling" of the radius. That is exactly double, not 1.775 rounded up or 2.00001 rounded down.
  - Therefore every integer value in a formula has an **infinite number of significant digits**.

Rounding a number to maintain significance

When we calculate a value or simply try to report a number with the proper number of significant digits we need to be able to round the number off. While this appears to be pretty straight forward, as always seems to be the case, there are a few exceptions.

Here are the accepted rules for rounding:

RULE #1: If the digit after the number of digits you are rounding to is greater than 5, then you round that final significant digit **UP.**

(eg) rounding 123.4**6** to 4 significant digits would yield: 123.5

RULE #2: If the digit after the number of digits you are rounding to is less than 5, then you round that final significant digit **DOWN**.

(eg) rounding 123.4**4** to 4 significant digits would yield: 123.4

RULE #3: If the digit after the number of digits you are rounding to is 5, then you must follow the **"odd-up rule".**

WHAT'S THE "ODD-UP RULE"????

Since we have established that the digit "0" is not significant, 1-4 (4 digits) round down and 6-9 (4 digits) round up. As a result, the digit "5" would need to be equally split between rounding up and rounding down in order to maintain equitability in the rounding process.

- If we round UP half of the time and DOWN the other half of the time, then any errors in rounding will eventually cancel each other out.

So, if the digit you are considering in your rounding is exactly "5" (no other digits following), then you:

- **ROUND UP, only if the digit to be rounded is ODD.**
- **ROUND DOWN, only if the digit to be rounded is EVEN.**

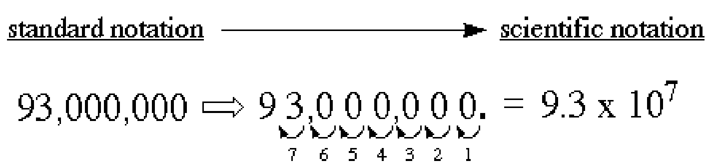
(eg) Rounding 123.**<u>4</u>**5 to 4 significant digits.
4 is an even number followed by a 5 – we round down: **123.4**

(eg) Rounding 123.**<u>7</u>**5 to 4 significant digits.
7 is an odd number followed by a 5 – we round up: **123.8**

(eg) Rounding 123.**<u>4</u>**501 to 4 significant digits.
4 is an even number followed by a 5, BUT with the extra "01" after the 5,
we are already **past the halfway point** and we round up: **123.5**

Hopefully, you have heard of scientific notation?

In chemistry, we often deal with numbers that are **very large** or **very small**. It is difficult/inconvenient to write so many zeros, and sometimes ***IMPOSSIBLE*** to write the correct number of significant figures. (eg. The number 10 cannot be written with 2 significant digits). We, therefore, need scientific notation to **move the decimal place** in the number in order to "change" it into a number between 1-10.



Scientific Notation must,

- ✓ Have **no more than 1 significant digit** before the decimal place
- ✓ Be multiplied by a **factor of ten with an exponent** that represents the number of spaces that the decimal place has been moved.
  - The **sign** of the exponent reveals the direction that the decimal needs to be moved in order to RETURN to the original number (standard notation).
    - **Positive exponent**, move the decimal to the right (number gets bigger)
    - **Negative exponent**, move the decimal to the left (number gets smaller)

NOTE: When changed into scientific notation, the number written MUST maintain the same number of significant figures!

For Example,

i. 0.00000004454 ➡ 4.454×10^{-8}
ii. 1010 ➡ 1.010×10^{4}
iii. 10 with 2 significant digits ➡ 1.0×10^{1}
iv. 75,000 with 4 significant digits ➡ 7.500×10^{4}

PRACTICE - TRY SOME FOR YOURSELF

(a) 1703 ➡ ____________________
(b) 0.057 ➡ ____________________
(c) 9000 ➡ ____________________
(d) 8.75×10^{-3} ➡ ____________________
(e) 1.1000×10^{2} ➡ ____________________
(f) 1 ➡ ____________________

Now for the hard part…

CALCULATIONS WITH SIGNIFICANT FIGURES

If 1 person has a mass of 64.5 kg, two people have a mass of 2 × 64.5 kg = 129 kg (3 significant digits)
If 1 book has a length of 3.5 cm, two books **DO NOT** have a length of 2 × 3.5 cm = 7 cm (1 significant digit)

When performing a calculation with significant digits being considered, your answer must maintain the accuracy and precision that is provided by the instruments being used, or the values provided.

Consider the volume of liquid in a graduated cylinder (as seen above) to be 30.1 mL, and a measurement of the mass of the same liquid to be 25.98 g. What would be the density of the liquid?

m = 25.98 g
V = 30.1 mL
D = ?

$$D = \frac{m}{V} \qquad D = \frac{25.98g}{30.1mL}$$

You will find that a good quality calculator will give you… D = 0.86312292358803986710963455149502 g/mL

- Does that seem right to you? How many digits would you keep? 3? 4? 6?
- Recall that you **cannot report more significant digits than your instruments can measure**.
- ***You can only be as certain as our poorest measuring device***
  (Think… "a chain is only as strong as its weakest link)

Depending on the type of mathematical operation being performed, there are 2 RULES for dealing with significant digits in a calculation:

1) Multiplication and/or Division

   The final answer of any multiplication or division will have the same number of <u>significant digits</u> as the starting value with the least number of significant digits.

   From our collected data we can see that our mass has 4 significant digits, and our volume has 3. Therefore, our answer must have 3 significant digits.

   Our density, then, must be 0.863 g/mL.

2) Addition and/or Subtraction

   The final answer of any addition or subtraction will have the same number of <u>decimal places</u> as the starting value with the least number of decimal places.

   So, if we were to subtract the mass of a 54.64 g graduated cylinder from the mass of the graduated cylinder full of water (let's say it's 89.3 g), our calculations might look something like this:

   $m_{water} = m_{full\ cylinder} - m_{empty\ cylinder}$
   = 89.3 g – 54.64 g — 1 decimal place minus 2 decimal places
   = ~~34.66~~ 34.7 g — Therefore, answer must be rounded so as to have **ONLY 1 decimal place**!

Sounds like a lot of hard work, eh?

Well, it is – welcome to chemistry!

Just like most things in this course… don't let yourself get frustrated too quickly and keep practicing. Hard work always pays off in the end!